Hall's harem theorem with additional properties

The Hall's harem theorem describes a condition which is equivalent to the existence of a perfect $(1, k)$-matching of a locally finite bipartite graph $\Gamma = (U, V, E)$. It is useful in amenability, for example to obtain some versions of Tarski's alternative theorem. In these applications, we use bipartite graphs where both U and V are copies of the same set. In such situation the matching realizes a k to 1 function, say $f : U \to U$.

Our work in computable amenability showed that to obtain computable versions of some results related to Tarski's alternative theorem it is necessary to find a perfect $(1, k)$-matching which realizes a function f with some additional properties. In particular we are interested in fast reaching of cycles of f.

In this talk I will show the conditions necessary for obtaining of such a matching and the corresponding construction.