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How to paint with tables? – A guide to the Builder-Painter game

Given two graphs G and H, a size Ramsey game is played on the edge set of $K_{\mathbb{N}}$. In every round, Builder selects an edge and Painter colours it red or blue. Builder's goal is to force Painter to create a red copy of G or a blue copy of H as soon as possible. The online (size) Ramsey number $\tilde{r}(G, H)$ is the number of rounds in the game provided Builder and Painter play optimally.

Let P_n denote a path with n vertices. Cyman, Dzido, Lapinskas and Lo [2] proved $\tilde{r}(P_3, P_n) = \lceil 1.25(n-1) \rceil$, $\tilde{r}(P_4, P_n) = 1.4n + O(1)$, $\tilde{r}(P_5, P_n) \geq 1.5(n-1)$ and conjectured, that $\tilde{r}(P_k, P_n) = 1.5n + o(n)$ for any fixed $k \geq 5$. Recently, Mond and Portier [3] disproved that hypothesis by showing that $\tilde{r}(P_{10}, P_n) \geq 1.(6)n - 2$. This matches (up to a constant for a fixed k) the upper bound $\tilde{r}(P_k, P_n) \leq 1.(6)n + 12k$ found by Bednarska-Bzdęga [1]. We improve the result of [3] by showing $\tilde{r}(P_9, P_n) \geq 1.(6)n - 2$. We also show that $\tilde{r}(P_8, P_n) \geq 1.(63)n - 2$ and $\tilde{r}(P_7, P_n) \geq 1.6n - 2$, therefore disproving the conjecture from [2] for $k \geq 7$. Our approach unifies methods used in [2] and [3].

This is joint work with Grzegorz Adamski.

References

[1] Małgorzata Bednarska-Bzdęga, *Off-diagonal online size Ramsey numbers for paths*, European Journal of Combinatorics (2024), Vol. 118, `arXiv:2310.09377`

[2] Joanna Cyman, Tomasz Dzido, John Lapinskas, Allan Lo, *On-line Ramsey numbers for paths and cycles*, Electron. J. Comb. **22** (2015) no. P1.15. `doi:10.37236/4097`.

[3] Adva Mond, Julien Portier, *The asymptotic of off-diagonal online Ramsey numbers for paths*, preprint, `arXiv:2312.16628`.