

Full complexity classification for list homomorphisms from bounded-treewidth graphs to reflexive graphs

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Streszczenie

In the *list homomorphism* problem, the input consists of two graphs G and H , together with a list $L(v) \subseteq V(H)$ for every vertex $v \in V(G)$. The task is to find a homomorphism $\phi : V(G) \rightarrow V(H)$ respecting the lists, that is, we have that $\phi(v) \in L(v)$ for every $v \in V(G)$ and if u and v are adjacent in G , then $\phi(u)$ and $\phi(v)$ are adjacent in H . If H is a fixed graph, then the problem is denoted by $\text{LHOM}(H)$. We consider the *reflexive* version of the problem, where we assume that every vertex in H has a self-loop. It is known that reflexive $\text{LHOM}(H)$ is polynomial-time solvable if H is an interval graph and it is NP-complete otherwise [Feder and Hell, JCTB 1998].

We explore the complexity of the problem parameterized by the treewidth $tw(G)$ of the input graph G . If a tree decomposition of G of width $tw(G)$ is given in the input, then the problem can be solved in time $|V(H)|^{tw(G)} \cdot n^{O(1)}$ by naive dynamic programming. Our main result completely reveals when and by exactly how much this naive algorithm can be improved. We introduce a simple combinatorial invariant $i^*(H)$, which is based on the existence of certain decompositions and incomparable sets, and show that this number should appear as the base of the exponent in the best possible running time. Specifically, we prove for every non-interval reflexive graph H that

- If a tree decomposition of width $tw(G)$ is given in the input, then the problem can be solved in time $i^*(H)^{tw(G)} \cdot n^{O(1)}$.
- Assuming the Strong Exponential-Time Hypothesis (SETH), the problem cannot be solved in time $(i^*(H) - \epsilon)^{tw(G)} \cdot n^{O(1)}$ for any $\epsilon > 0$.

Thus by matching upper and lower bounds, our result exactly characterizes for every fixed H the complexity of reflexive $\text{LHOM}(H)$ parameterized by treewidth.

The results are obtained as a joint work with Dániel Marx and László Egri.