SHUFFLE SQUARES AND NEST-FREE GRAPHS

JAROSŁAW GRYTCZUK, BARTŁOMIEJ PAWLIK, AND ANDRZEJ RUCIŃSKI

ABSTRACT. A *shuffle square* is a word consisting of two shuffled copies of the same word. For instance, the French word `tuteurer` is a shuffle square, as it can be split into two copies of the word `tuer`. An *ordered graph* is a graph with a fixed linear order of vertices.

We propose a representation of shuffle squares in terms of special nest-free ordered graphs and demonstrate the usefulness of this approach by applying it to several problems. Among others, we prove that binary words of the type $(ABBA)^n$, n odd, are not shuffle squares and, moreover, they are the only such words among all binary words whose every A-run has length one or two, while every B-run has length two.

We also provide a counterexample to a believable stipulation that binary words of the form $1^n 0^{n-2} 1^{n-4} 0^{n-6} \ldots$, n odd, are far from being shuffle squares (the distance measured by the minimum number of letters one has to delete in order to turn a word into a shuffle square).

FACULTY OF MATHEMATICS AND INFORMATION SCIENCE, WARSAW UNIVERSITY OF TECHNOLOGY, 00-662 WARSAW, POLAND
Email address: `jaroslaw.grytczuk@pw.edu.pl`

INSTITUTE OF MATHEMATICS, SILESIAN UNIVERSITY OF TECHNOLOGY, 44-100 GLIWICE, POLAND
Email address: `bpawlik@polsl.pl`

FACULTY OF MATHEMATICS AND COMPUTER SCIENCE, ADAM MICKIEWICZ UNIVERSITY, 61-614 POZNAŃ, POLAND
Email address: `rucinski@amu.edu.pl`