Complexity of H-coloring on graphs with bounded treewidth

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03.12.2019

Abstract

For graphs G and H, a homomorphism from G to H is an edge-preserving mapping from the vertex set of G to the vertex set of H. For a fixed graph H, the $\textsc{Hom}(H)$ problem asks if an instance graph G on n vertices admits a homomorphism to H. If H is a complete graph with k vertices, then $\textsc{Hom}(H)$ is equivalent to the k-$\textsc{Coloring}$ problem, so graph homomorphisms can be seen as generalizations of colorings.

If a tree decomposition of G of width t is given in the input, then the problem can be solved in time $|V(H)|^t \cdot poly(n)$ by dynamic programming. I will show for which H the existence of a significantly faster algorithm is unlikely. Also, I will talk about cases in which we were not able to prove a tight bound for the complexity and how it relates to big open problems in the algebraic graph theory.

This is based on a joint work with Paweł Rzążewski.