# Fractional Local Dimension

Tom Trotter (Georgia Institute of Technology)

19th of June 2018

**Abstract**

The original notion of dimension for posets was introduced by Dushnik and Miller in 1941 and has been studied extensively in the literature. In 1992, Brightwell and Scheinerman developed the notion of fractional dimension as the natural linear programming relaxation of the Dushnik-Miller concept. In this paper, we introduce and study a new parameter for posets we call *fractional local dimension*. As suggested by the terminology, our parameter is a common generalization of *fractional dimension* and *local dimension*.

For a pair $(n, d)$ with $2 \leq d < n$, we consider the poset $P(1, d; n)$ consisting of all 1-element and $d$-element subsets of $\{1, 2, \ldots, n\}$ partially ordered by inclusion. This poset has fractional dimension $d + 1$, but for fixed $d \geq 2$, its local dimension goes to infinity with $n$. On the other hand, we show that as $n$ tends to infinity, the fractional dimension of $P(1, d; n)$ tends to a value $f(d)$ which asymptotically satisfies $f(d) = d/(\log d - \log\log d - o(1))$. However, for all $d \geq 2$, we will be able to determine $f(d)$ *exactly*. As an immediate corollary, we show that if $P$ is a poset and $d$ is the maximum degree of a vertex in the comparability graph of $P$, then the fractional local dimension of $P$, is at most $2 + f(d)$.

Our arguments use both discrete and continuous methods.

This is work co-authored with Heather Smith.