Sparse Kneser graphs are Hamiltonian

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Streszczenie

For integers $k \ge 1$ and $n \ge 2k + 1$, the Kneser graph K(n, k) is the graph whose vertices are the k-element subsets of $\{1, \ldots, n\}$ and whose edges connect pairs of subsets that are disjoint. The Kneser graphs of the form K(2k + 1, k) are also known as the odd graphs. We settle an old problem due to Meredith, Lloyd, and Biggs from the 1970s, proving that for every $k \ge 3$, the odd graph K(2k + 1, k) has a Hamilton cycle. Its construction is based on constructing a spanning tree in a suitably defined hypergraph on Dyck words. As a byproduct, it provides an alternative proof of the so-called middle levels conjecture, originally proved by Mütze in 2014.

Joint work with Torsten Mütze and Jerri Nummenpalo: arXiv:1711.01636.