A k-uniform hypergraph H is ℓ-Hamiltonian saturated, $1 \leqslant \ell \leqslant k-1$, if H does not contain an ℓ-overlapping Hamiltonian cycle $C_n^{(k)}(\ell)$ but every hypergraph obtained from H by adding one more edge does contain $C_n^{(k)}(\ell)$. Let $sat(n,k,\ell)$ be the smallest number of edges in an ℓ-Hamiltonian saturated k-uniform hypergraph on n vertices. Clark and Entringer proved in 1983 that $sat(n,2,1) = \lceil \frac{3n}{2} \rceil$ and the second author showed recently that $sat(n,k,k-1) = \Theta(n^{k-1})$. In this paper we prove that $sat(n,k,\ell) = \Theta(n^{\ell})$ for $\ell = 1$ as well as for all $\ell \geqslant 0.8k$.