

Partition regularity of homogeneous systems of polynomial equations

Abstract: A polynomial equation $p(x_1, \dots, x_n) = 0$ is partition regular if for any partition $\mathbb{N} = \bigcup_{i=1}^r C_i$, there is some cell C_{i_0} of the partition containing a solution to the given equation. An active area of research in modern Ramsey theory is determining which polynomial equations are partition regular. While there are many known examples of partition regular polynomial equations such as $x - y = z^2$ and $z = xy + x + y$, there are still many simple equations such as $x^2 - y^2 = z^2$ and $z = xy + x$ whose partition regularity remains unknown. In this talk, we will examine the partition regularity of some systems of polynomial equations such as

$$\begin{aligned} wz &= x_1^2 - y_1^2 & \text{and} & & w_1 z_1 &= x^2 - y^2 \\ 2wz &= x_2^2 - y_2^2 & & & 2w_2 z_2 &= x^2 - y^2 \end{aligned} ,$$

one of which is partition regular while the other is not.