Random regular r-graphs: counting, coupling and cycles

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Abstract

An r-uniform hypergraph on a vertex set V is a family of r-element subsets (called edges) of V. A hypergraph is d-regular, if every vertex belongs to exactly d edges. In the case of graphs ($r = 2$), a general asymptotic formula (as $n = |V|$ grows) for the number of of d-regular graphs was obtained by McKay and Wormald in 1991. We extend their approach to obtain a formula for every $k > 2$. In addition, the method allows us to show the following fact, provided that d grows faster than log n but slower than $n^{1/2}$. If $H(n, d)$ is a random d-regular hypergraph, then with high probability it contains $H(n, m)$, the random (not necessarily regular) hypergraph with exactly m edges.

As a consequence, we can transfer some known asymptotic properties of the graph $H(n, m)$ to the graph $H(n, d)$. For example, we get that with high probability H(n,d) contains a loose Hamilton cycle. (Joint work with A. Dudek, A. Frieze and A. Ruciski.)