

Abstract: Consider a directed graph D whose vertices are colored by real numbers. Every vertex computes its own *value*: the sum of incoming colors minus the sum of outgoing colors (or vice versa). A coloring of D is *non-vanishing* if the value of every vertex is non-zero. How many colors are needed for a non-vanishing coloring of D ? We conjecture that every digraph D having a *perfect matching* has a non-vanishing coloring by just *three* real numbers (for instance, 1, 2, 3). This statement implies the famous 1-2-3 Conjecture and is implied by a more general conjecture of Niwczyk, which in turn is a special case of a conjecture of Saxton.

In a similar flavor, a coloring of a hypergraph H is *non-vanishing* if the sum of colors in every edge is non-zero. Trivially, every hypergraph has a non-vanishing coloring by just *one* real number (provided it is not zero), but the *list coloring* version of the problem is highly non-trivial. Suppose, for instance, that H is a *complete* hypergraph on n vertices (every non-empty subset is an edge in H). Goddyn and Tarsi conjecture that this hypergraph has a non-vanishing coloring from arbitrary lists of size $n + 1$, which is best possible. This problem has an intriguing connection to the Lonely Runner Problem, and also leads to the "non-vanishing" version of the famous Rota Basis Conjecture (stating that every matroid of rank n which is a disjoint union of n bases can be split into n *rainbow* bases).